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A CLASS OF TRAVEL DEMAND ESTIMATORS USING TRAFFIC FLOWS

by

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ABSTRACT

In this paper a class of travel demand parameter estimators is proposed, based on the asumption that correct statistical use of different information sources produces "better" results. Starting from this postulate, different estimators are obtained, both for demand model parameters and O-D matrix, by properly combining "ad hoc" survey results, "a priori" information and observed traffic flows. All estimators, which are GLS or ML when exclusively based on experimental results and Bayesian otherwise, can be obtained by solving an extremum problem with a quadratic in the demand objective function.

Furthermore it is shown that under reasonable hypotheses the use of traffic flows results in an improvement of demand parameter estimates even when only an approximate assignment model is adopted, provided that due allowance is made for assignment errors.

I - INTRODUCTION

A large body of literature now exists dealing with travel demand estimation methods. These methods can be roughly divided into two categories depending on the information sources they use.

A firt group encompasses methods based on "ad hoc" sampling surveys such as home or destination interviews. "Direct" sampling estimators of travel demand and models calibrated on survey results belong to the first group.

A second and more recent group comprises demand estimators uniquely based on traffic flow measures. These, in turn, can be subdivided into model parameter estimators and in direct O-D matrix estimators from traffic counts. These methods recently received considerable attention because of their evident practical convenience.

Statistical hypotheses underlying the proposed estimators, however, are seldom made explicit. For instance no explicit reference is made to the estimators properties as a consequence of the starting O-D matrix "quality" or to biases introduced by assignment model mispecification; nor has the possibility of using other information sources been considered. Only recently were results overcoming some of these inconveniences for O-D matrix estimation presented, Cascetta (1983, 1984), Maher (1984).

In this paper a more general approach to the estimation problem is proposed, based on the assumption that correct statistical use of all available information sources produces "better" estimates of unknown parameters. Starting from this postulate a class of different estimators can be obtained by properly combining "ad hoc" survey results, "a priori" information and flow measures to estimate both model parameters and direct O-D matrices.

In order to improve estimates, however, information sources have to be correctly used. This means, for instance, that when the relationship between observed flows and travel demand is used via an assignment model, allowance must be made for errors deriving from measurement, temporal variations of demand and model's inability to fully represent reality. In the paper it is shown that, under reasonable hypotheses, this can be at least partially accomplished by considering that informations contained in flows are intrinsically stochastic. In the next section information about travel demand contained in ad hoc surveys and traffic flow measures are analyzed and formally stated.

In section 3 two estimators for demand model parameters are proposed : a two stage GLS or ML estimator combining a survey based estimate of parameters with observed flows and a single stage GLS estimator using a sample O-D matrix estimate with traffic counts.

In section 4 the estimation of model parameters in the case in which no survey results are available is considered. Here again, two estimators are analysed : a GLS estimator based on traffic flows only and a Bayesian estimator combining "a priopri" knowledge about parameters with observed flows.

In section 5 GLS and Bayesian estimators for O-D matrix proposed in literature are obtained as particular cases of previously considered parameter estimators.

Finally, in section 6, implementation of the proposed estimators is considered and problems deriving from the use of equilibrium traffic assignment models and the use of information in the form of inequality constraints are analyzed.

II - INFORMATIONS ABOUT TRAVEL DEMAND CONTAINED IN AD HOC SURVEYS AND TRAFFIC FLOW MEASURES

Let us suppose that the trips made in a reference period by using available transportation modes between pairs of origin-destination nodes are ordered along the n components of a column vector $\underline{t}$. Every component of $\underline{t}$ will thus give the number of trips between a certain O-D pair with a certain mode. If we denote by $\underline{\tilde{t}}$ the estimate of multimodal O-D matrix obtained by a sample survey and if the estimator used is unbiased, it can be assumed that :

$$\underline{\tilde{t}} = \underline{t} + \underline{\zeta} \qquad (1)$$

$$E(\underline{\zeta})=\underline{0} \qquad E(\underline{\zeta}\,\underline{\zeta}')=\underline{Z}$$

The estimate variances-covariances matriz $\underline{Z}$ can be obtained on the basis of the sample extraction procedures and the adopted estimator by applying standard sampling theory results.

Furthermore, let us assume that a demand function relating the multimodal O-D matrix to exogenous variables via a K-dimensional parameters vector $\underline{\beta}$ is correctly specified. The assumed demand model can be seen as a vectorial function $\underline{t}\,(\underline{\beta})$ transforming the parameter space R^k into the O-D pairs space R^n_+.

In general the actual O-D matrix do not conform with that predicted by the model, even if we use the "true" vector $\underline{\beta}$. This is because the model is only an approximation to real phenomena underlying mobility; furthermore mobility, strictly speaking, should be seen as a stochastic process over homogeneous time periods with a mean that is usually assumed to be given by the model.

The above discussion can be summarized in the following :

$$\underline{t} = \underline{t}\,(\underline{\beta}) + \underline{\gamma}$$
$$E\,(\underline{\gamma}) = \underline{0} \qquad E\,(\underline{\gamma}\,\underline{\gamma}') = \underline{G} \tag{2}$$

where the dispersion matrix $\underline{G}$ of the random vector $\underline{\gamma}$ can be expressed inductively, for instance by assuming that O-D matrices in different periods are determinations of multinomial experiment with probability given by the model.

Alternatively one can obtain estimates of time variability and the model residuals directly from survey results. Systems of stochastic equations (1) and (2) can be combined obtaining :

$$\bar{\underline{t}} = \underline{t}\,(\underline{\beta}) + \underline{\mu}$$
$$E(\underline{\mu}) = \underline{0} \qquad E(\underline{\mu}\,\underline{\mu}') = \underline{Z} + \underline{G} = \underline{M} \tag{3}$$

where the random vector $\underline{\mu}$ is sum of the two vectors $\underline{\zeta}$ and $\underline{\gamma}$ and its dispersion matrix $\underline{M}$ is the sum of their dispersion matrices $\underline{Z}$ and $\underline{G}$ under the acceptable hypothesis that $\underline{\zeta}$ and $\underline{\gamma}$ are independently distributed.

Let us now analyze another possible source of information : arc flows.

Without any loss of generality we can assume that flows passing over network m arcs in the reference period are related to the true O-D matrix relative to the same period by the equation :

$$\underline{f} = \underline{A}\,\underline{t} \tag{4}$$

where the (m x n) assignment matrix $\underline{A}$ gives the fractions of different O-D pairs making up the true flows.

In general we do not know the true assignment matrix $\underline{A}$, but only with an estimate $\bar{\underline{A}}$ obtained by using one of the available assignment models.[1] This implies that even if we knew the true O-D matrix $\underline{t}$ and assigned it to the network, we would obtain a vector of arc flows differing from the true one :

$$\bar{\underline{f}} = \bar{\underline{A}}\,\underline{t} \qquad (5)$$

Discrepencies between $\bar{\underline{f}}$ and $\underline{f}$ can be due to many factors : temporal variations of demand, variations in paths use proportions with respect to the average value given by the assignment model and assignment and network models mispecifications.

It is reasonable to assume that the "network extraction" component contributes to a systematic error, i.e. an error which does not cancel out among different links, and that this error could be reduced by a more detailed representation of the network. On the other hand all others errors should tend to be distributed randomly around a zero mean, at least for links belonging to homogeneous categories.

Both these assumptions receive a strong experimental support from the results of Bovy and Jansen (1983). The authors reach the conclusion that even a medium level of network aggregation, i.e. considering only primary and secondary roads, reduces the systematic bias of the assigned flows to approximately 10% of the mean. Furthermore they suggest that a correction procedure could be set up as a result of empirical studies even for a coarser level of network representation.

In conclusion, if the systematic bias due to spatial aggregation can be ignored, both because negligible or eliminated, we can assume that differences between true and assignment flows over the considered arcs are determinations of random variables with zero mean so that :

$$\underline{f} = \bar{\underline{A}}\,\underline{t} + \underline{\delta} \qquad (6)$$

$$E(\underline{\delta}) = \underline{0} \qquad E(\underline{\delta}\,\underline{\delta}') = \underline{D}$$

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[1] In the following the "assignment operator" will be denoted by $\bar{\underline{A}}$. If a proportional assignment model is adopted the operator is a matrix whose elements are independent of the O-D vector. In the case of equilibrium models, deterministic or stochastic, $\bar{\underline{A}}$ can still be considered a matrix with elements depending on the O-D vector assigned via path costs.

Dispersion matrix $\underline{D}$ depends on observed link types, the chosen assignment model and the particular network structure. In fact model errors seem to have different variances for different link types, see Bovy and Jensen (1983), furthermore a network structure allowing many alternative routes and a more unrealistic assignment model results in higher residual dispersion and this, in turn, implies worse estimates of demand parameters as will be shown in the next section. From a practical point of view the dispersion matrix $\underline{D}$ can be obtained by experimental results for deterministic models and from path choice probabilities for stochastic assignment models, Daganzo (1977). When the model based O-D matrix $\underline{t}\ (\underline{\beta})$ is substituted in expression (6) and allowance for measurement errors in flows is made we obtain[1] :

$$\underline{\bar{f}} = \underline{\bar{A}}\ \underline{t}\ (\underline{\beta}) + \underline{v}$$

$$E\ (\underline{v}) = \underline{0} \qquad E(\underline{v}\ \underline{v}') = \underline{D} + \underline{\bar{A}}\ \underline{G}\ \underline{\bar{A}}' + \underline{T} = \underline{W} \tag{7}$$

where $\underline{\bar{f}}$ is the vector of measured flows and $\underline{T}$ the dispersion matrix of measurement errors.

III - ESTIMATION OF DEMAND MODEL PARAMETERS COMBINING SAMPLE SURVEYS AND TRAFFIC FLOWS

In this section two estimators for the parameters of a travel demand model will be proposed, both using surveys and traffic flows as information sources.

The first estimator can be considered as a two-stage one. In fact if a first stage estimate of $\underline{\bar{\beta}}$ of the parameters vector $\underline{\beta}$ was obtained by using sampling survey results with an unbiased estimator, it can be assumed that :

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[1] Expression (7) for the dispersion matrix $\underline{W}$ is strictly correct in the case of proportional assignment models, e.g. all or nothing or stochastic network loading. If an equilibrium model is used it can give only a rough approximation by calculating fractions of O-D pairs loading different links at equilibrium. Alternatively empirical expressions giving variances as function of link volumes can be used.

$$\bar{\underline{\beta}} = \underline{\beta} + \underline{\varepsilon}$$
$$E(\underline{\varepsilon}) = \underline{0} \qquad E(\underline{\varepsilon}\,\underline{\varepsilon}') = \underline{V} \tag{8}$$

Approximate expressions for the estimator dispersion matrix $\underline{V}$ can be obtained, as a function of the estimation method used, see for instance Domencich and McFadden (1975) and Daganzo (1979) for disaggregate models and Giles and Hampton)1981) for gravity models.

Systems of stochastic equations (7) and (8) can be combined giving rise to a K+m equation system relating "observed" vectors $\bar{\underline{\beta}}$ and $\bar{\underline{f}}$ to the unknown parameter vector $\underline{\beta}$:

$$\bar{\underline{\beta}} = \underline{\beta} + \underline{\varepsilon}$$
$$\bar{\underline{f}} = \bar{\underline{A}}\,\underline{t}(\underline{\beta}) + \underline{v} \tag{9}$$

Under the assumption of null or negligible covariances among components of vectors $\underline{\varepsilon}$ and $\underline{v}$, the mean and dispersion matrix of residual vector $(\underline{\varepsilon}', \underline{v}')$ can be expressed as :

$$E(\underline{\varepsilon}'\ \underline{v}') = \underline{0} \qquad E\left(\begin{bmatrix}\underline{\varepsilon}\\ \underline{v}\end{bmatrix}\begin{bmatrix}\underline{\varepsilon}\\ \underline{v}\end{bmatrix}'\right) = \begin{bmatrix}\underline{V} & \underline{0}\\ \underline{0} & \underline{W}\end{bmatrix} \tag{10}$$

If no distributional assumptions are made about the random vectors $\underline{\varepsilon}$ and $\underline{v}$, parameter vector $\underline{\beta}$ can be estimated by using a nonlinear Generalized Least Square (GLS) estimator $\underline{\beta}_1$, which, in turn, is the solution of the extremum problem :

$$\min_{\beta} S_1(\beta) = \min\ (\beta-\bar{\underline{\beta}})'\underline{V}^{-1}(\underline{\beta}-\bar{\underline{\beta}}) + (\bar{\underline{f}}-\underline{A}\underline{t}(\underline{\beta}))'\underline{W}^{-1}(\bar{\underline{f}}-\underline{A}\underline{t}(\underline{\beta})) \tag{11}$$

Statistical properties of non-linear GLS estimnators are only asymptotic, it can be shown that they are consistent and asymptotically normally distributed. Furthermore an approximate expression for the dispersion matrix $\underline{B}_1$ of $\underline{\beta}_1$ can be obtained, Judge et al. (1980)[1], as :

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[1] Here and in the following dispersion matrices of proposed estimators are calculated in the hypothesis that the assignment matrix $\underline{A}$ is known. These expressions are also valid in the equilibrium assignment case conditional to the resulting matrix $\bar{A}$, so that they still serve their main purpose, i.e. to show the variance reduction effect when traffic flows are used.

$$\underline{B}_1 \doteq [\underline{V}^{-1} + \underline{J}\,(\underline{\beta}_1)'\bar{\underline{A}}'\underline{W}^{-1}\,\bar{\underline{A}}\,\underline{J}\,(\underline{\beta}_1)]^{-1} \qquad (12)$$

where $\underline{J}\,(\underline{\beta}_1)$ is the Jacobian matrix of $\underline{t}(\underline{\beta})$ evaluated at $\underline{\beta}_1$.

Expression (12) can be put in an alternative form as :

$$\underline{B}_1 \doteq \underline{V} - \underline{V}\,\underline{J}\,(\underline{\beta})'\,\bar{\underline{A}}'\,[\underline{W} + \bar{\underline{A}}\,\underline{J}\,(\underline{\beta})\,\underline{V}\,\underline{J}\,(\underline{\beta})'\,\bar{A}']^{-1}\,\bar{\underline{A}}'\,\underline{J}\,(\underline{\beta})\,\underline{V} \qquad (13)$$

Expression (13) shows that the use of informations contained in traffic flows reduces the variances of model parameter estimates. In fact the dispersion matrix of $\underline{\beta}_1$ is the difference between the starting estimate $\bar{\underline{\beta}}$ dispersion matrix $\underline{V}$ and a positive definite matrix. In turn, this implies that the variance of each component of $\underline{\beta}_1$ is lower than that of the analogous component of $\bar{\underline{\beta}}$; although quantitative evaluation of the variance reduction effect has to be made case by case from expression (13), it can be said that the reduction will be the larger the lower the dispersion of $\underline{v}$, matrix $\underline{W}$, and the larger the assignment matrix rank. To put it in another way the more independent flow information we use and the more precise this information is, the better $\underline{\beta}$ estimate we get. This conclusion will be quantitatively confirmed by the numerical analysis of small gravity model reported in the appendix. It is often reasonable to assume that random vectors $\underline{\varepsilon}$ and $\underline{v}$ of equation (9) follow independent multivariate normal distributions as most estimators giving $\bar{\underline{\beta}}$ are asimptotically normally distributed and the many additive components of the residual vector $\underline{v}$ can be assumed to produce a normal variate, at least for large enough flow values.

It is well known that under a normality assumption, nonlinear GLS and Maximum Likelihood estimators coincide so that in this case also the ML estimators asymptotic efficiency can be credited to $\underline{\beta}_1$. Numerical computations of $\underline{\beta}_1$ can be performed by using one of the many general-purpose non constrained minimization algorithms or else one of the algorithms especially designed for nonlinear GLS, such as Gauss-Method ones, can be

used. As anticipated at the beginning of this section, a different estimator using sample surveys and observed flows can be obtained.

This second estimator $\underline{\beta}_2$ can be seen as a single stage one. In fact we can combine equations (3) and (7) expressing the relationship between the observed O-D matrix estimate $\underline{\bar{t}}$ and arc flows $\underline{\bar{f}}$ and the model's parameter vector $\underline{\beta}$:

$$\begin{aligned} \underline{\bar{t}} &= \underline{t}\,(\underline{\beta}) + \underline{\mu} \\ \underline{\bar{f}} &= \underline{\bar{A}t}\,(\underline{\beta}) + \underline{\nu} \end{aligned} \tag{16}$$

In general random vector $\underline{\mu}$ and $\underline{\nu}$ are not independent insomuch as both are linear function of $\underline{\gamma}$, as results from (3) and (7). Mean and dispersion matrices of $(\underline{\mu}',\underline{\nu}')$ are given by :

$$E\,(\underline{\mu}',\underline{\nu}') = \underline{0}' \qquad E\left(\begin{bmatrix}\underline{\mu}\\ \underline{\nu}\end{bmatrix}\begin{bmatrix}\underline{\mu}\\ \underline{\nu}\end{bmatrix}'\right) = \begin{bmatrix}\underline{M} & \underline{AG}\\ \underline{G}'\underline{A}' & \underline{W}\end{bmatrix} \tag{17}$$

When dispersion of the model residuals is considerably lower than that of sampling errors, it is possible to ignore $\underline{\mu}$ and $\underline{\nu}$ covariances and consider dispersion matrix (17) as block diagonal. The GLS estimation principle can be adopted again to obtain the estimator $\underline{\beta}_2$, which, under the above assumptions, is the solution of the minimum problem :

$$\min_{\beta} S\,(\underline{\beta}) = \min\,\{(\underline{\bar{t}} - \underline{t}(\underline{\beta}))'\,\underline{M}^{-1}(\underline{\bar{t}}-\underline{t}(\underline{\beta})) + (\underline{\bar{f}}-\underline{\bar{A}t}(\underline{\beta}))'\,\underline{W}^{-1}(\underline{\bar{f}}-\underline{\bar{A}t}(\underline{\beta})) \tag{18}$$

An approximate expression of the $\underline{\beta}_2$ dispersion matrix $\underline{B}_2$ can be obtained as :

$$\underline{B}_2 = [\underline{J}'(\underline{\beta}_2)\,\underline{M}^{-1}\,\underline{J}(\underline{\beta}_2) + \underline{J}(\underline{\beta}_2)'\,\underline{\bar{A}}'\,\underline{W}^{-1}\,\underline{\bar{A}}\,\underline{J}\,(\underline{\beta}_2)^{-1} \tag{19}$$

From (19) it can be seen once again that the use of traffic flow information improves the $\underline{\beta}$ estimates insomuch as $\underline{\beta}_2$ has lower variance than the GLS estimator obtained by using only survey results via equation (3). In fact this latter estimator has a dispersion matrix given by the inverse of the first part of second member of expression (19). In this sense estimator $\underline{\beta}_2$ can be considered as a generalization of nonlinear LS estimators

proposed in literature, e.g. for gravity models Kirby (1974), Cesareo (1975), or for logit models, Domencich and McFadden (1975). It can be noted from (12) and (19) that the single stage estimator dispersion matrix $\underline{B_2}$ tends to coincide with the two stage estimator dispersion matrix $\underline{B_1}$ when a GLS technique is used to obtain the starting estimate $\underline{\bar{\beta}}$.

Comparison of statistical performances of $\underline{\beta_1}$ and $\underline{\beta_2}$ should be carried out numerically on a case by case basis. From a general point of view, it can be said that while $\underline{\beta_2}$ is computationally easier to obtain not requiring the previous estimation of $\underline{\bar{\beta}}$, $\underline{\beta_1}$ can use an estimator for $\bar{\beta}$ which is statistically more efficient than a GLS one and is amenable to an ML interpretation with the resulting properties.

IV - ESTIMATION OF DEMAND MODEL PARAMETERS FROM TRAFFIC FLOWS

In the case in which the only available experimental data are traffic flows it is still possible to obtain estimators for the parameters vector $\underline{\beta}$. In this section two such estimators are considered; one obtained by following a classic sampling approach, of the type considered in the preceding section, and a second deriving from a Bayesian approach. The first estimator $\underline{\beta_3}$ can be seen as a particular case of the estimators proposed in the previous section when no survey based information, vectors $\underline{\bar{\beta}}$ or $\underline{\bar{t}}$, is available. In this case when the number of observed flows is greater than that of $\underline{\beta}$ components, a GLS, or ML under a flow normal distribution hypothesis, estimator $\underline{\beta_3}$ can be obtained by using only flow based equations (7) :

$$\min_{\beta} S_3(\underline{\beta}) = \min (\underline{\bar{f}} - \underline{\bar{A}t}(\underline{\beta}))' \underline{W}^{-1} (\underline{\bar{f}} - \underline{\bar{A}t}(\underline{\beta})) \qquad (20)$$

Estimators of this type have been proposed by many authors. Some of them considered a nonlinear specification of $\underline{t}(\underline{\beta})$, e.g. Robillard (1975), Hogberg (1976), while others adopted a linear model structure, e.g. Low (1973), Holm et al. (1976), Gaudry and Lamarre (1979).

Most proposed estimators, however, are Least Squares ones so that a GLS estimator explicitly considering residual heteroschedasticity and autocorrelation should have better statistical properties.

An approximate expression of the $\underline{\beta}_3$ dispersion matrix can be obtained by setting $\underline{V}$ equal to zero in expression (12).

It immediately results that the loss of sampling survey information causes an increase in the dispersion of $\underline{\beta}$ estimates and that the same effect is produced by more approximate assignment models.

When the planner has some "a priori" information about the parameter vector $\underline{\beta}$, such as values assumed in similar situations or ranges of possible variation for all or some of its components, it is sensible to make use of this information in conjunction with that contained in observed flows. The "natural" way to do so is to use Bayesian estimation theory.

If a multivariate normal with mean $\underline{\bar{\beta}}$ and dispersion matrix $\underline{V}$ is assumed as "a priori" probability density function for $\underline{\beta}$ and also observed flow distribution is assumed to be normal with mean $\underline{\bar{A}t}\ (\underline{\beta})$ and dispersion matrix $\underline{W}$, from Bayes theorem it results that the $\underline{\beta}$ probability density function (p.d.f.) conditional to the observed $\underline{\bar{f}}$ vector is

$$g(\underline{\beta}/\underline{\bar{f}}) \propto \exp\left[-(\underline{\beta}-\underline{\bar{\beta}})'\underline{V}^{-1}\ (\underline{\beta}-\underline{\bar{\beta}})\right] \cdot \exp\left[-(\underline{\bar{f}}-\underline{\bar{A}t}(\underline{\beta}))'\underline{W}^{-1}(\underline{\bar{f}}-\underline{\bar{A}t}(\underline{\beta})\right] \qquad (21)$$

Bayes estimator $\underline{\beta}_B$ is obtained as the vector maximizing (21) or, equivalentely, minimizing $S\ (\underline{\beta})$ as given by (11). It should be noted that similarities between the two stage estimator $\underline{\beta}_1$ and $\underline{\beta}_B$ are <u>purely formal</u> insomuch as in one case vector $\underline{\bar{\beta}}$ is a sample based estimate with dispersion matrix $\underline{V}$ while in the other it only represents the parameters "a priori" expected value which would be used by the planner lacking other information.

V - DIRECT O-D MATRIX ESTIMATION

For many practical applications knowledge of an O-D matrix is of more interest than a travel demand function.

It has also been widely recognized that traffic flows can help to give a better O-D matrix estimate, as the amount of literature on this subject shows. Also in the direct estimation of O-D matrix, two different situations can be distinguished depending on whether a sampling survey based estimate $\underline{\bar{t}}$ is available or not. Both these cases have reen recently covered in the literature, Cascetta (1983, 1984), Maher (1984), but with an interpretation of flow residuals somewhat different from that proposed in this paper.

In the following only a brief outline of the main results will be reported with the aim of demonstrating that direct O-D matrix estimators can be considered as special cases of those presented in preceding sections.

In fact the O-D estimation problem can be reduced to that of model parameter estimation by noting that in this case the vector $\underline{\beta}$ to be estimated coincides with the matrix itself $\underline{t}$:

$$\underline{t}\,(\underline{\beta}) \equiv \underline{\beta} \equiv \underline{t} \tag{22}$$

When sampling survey results are available, equations relating the estimated O-D matrix $\underline{\bar{t}}$ and arc flows $\underline{f}$ to the true matrix $\underline{t}$ via the adopted assignment model are given by expressions (1) and (6) respectively. These two systems can be combined together, giving rise, after allowance is made for measurement errors to :

$$\underline{\bar{t}} = \underline{t} + \underline{\zeta}$$
$$\underline{\bar{f}} = \underline{\bar{A}t} + \underline{v} \qquad (23)$$

$$E\,(\underline{\zeta}', \underline{v}') = 0' \qquad E\left(\begin{bmatrix}\underline{\zeta}\\ \underline{v}\end{bmatrix}\begin{bmatrix}\underline{\zeta}\\ \underline{v}\end{bmatrix}'\right) = \begin{bmatrix}\underline{Z} & \underline{0}\\ \underline{0} & \underline{N}\end{bmatrix}$$

Where the vector $\underline{v}$ is the sum of assignment errors $\underline{\delta}$ and measurement errors and its dispersion matrix $\underline{N}$ is the sum of their matrices.

The GLS-O-D vector estimator $\underline{t_1}$ is the solution of the extremum problem :

$$\min_t S_1(t) = \min_t (\underline{t}-\underline{\bar{t}})'\ \underline{Z}^{-1}\ (\underline{t}-\underline{\bar{t}}) + (\underline{\bar{f}}-\underline{\bar{A}t})'\ \underline{N}^{-1}\ (\underline{\bar{f}}-\underline{\bar{A}t}) \qquad (24)$$

which has a convex quadratic objective function if a proportional assignment is used. In this case an explicit solution can be obtained by equating the $S_1(t)$ gradient to zero :

$$\underline{t_1} = (\underline{Z}^{-1}+\underline{\bar{A}}'\underline{N}^{-1}\underline{\bar{A}})^{-1}\ (\underline{Z}^{-1}\underline{\bar{t}}+\underline{\bar{A}}'\underline{N}^{-1}\underline{\bar{f}}) \qquad (25)$$

Dispersion matrix $\underline{T_1}$ of $\underline{t_1}$ can also be explicitely obtained as :

$$\underline{T_1} = \underline{Z} - \underline{Z\bar{A}}'\ (\underline{N}\ \underline{\bar{A}Z\bar{A}}')^{-1}\ \underline{\bar{A}Z} \qquad (26)$$

demonstrating again that the use of traffic flows reduces starting estimate variances.

It can be easily verified that expression (24) and (26) coincide with the analogous (11) and (13) once that substitutions (22) are made. Specialization of $\underline{t_1}$ to the case of deterministic flows constraints, i.e. null $\underline{N}$ matrix and further results regarding the estimators statistical performances can be found in Cascetta (1983, 1984). It must however be realized that using a deterministically constrained estimator when the vector $\underline{v}$ in (23) is not null causes the resulting estimates to be biased and so no "a priori" statistical superiority with respect to starting estimates can be credited to them. For the direct estimation of an O-D

matrix in the absence of survey results, a Bayesian approach similar to the one followed for β_B can be used. In this case, if a multinormal is assumed both for the $\underline{t}$ "a priori" distribution, with mean $\underline{\bar{t}}$ and dispersion matrix $\underline{Z}$, and for the observed flows p.d.f. then the Bayes estimator $\underline{t_B}$ can be obtained by minimizing $S_1(\underline{t})$ and formally coincide with $\underline{t_1}$.

Most "mathematical programming " O-D matrix estimators proposed in the literature can be considered under a sampling or Bayesian framework even if hypotheses about the starting estimate $\underline{\bar{t}}$ are not explicitely stated. In particular in Cascetta (1984) and Maher (1984) some numerical results can be found suggesting that Maximum Entropy estimators (Van Zuylen and Willumsen, 1980) give results very similar to those obtained by $\underline{t_1}$ and $\underline{t_B}$ when the dispersion matrix $\underline{Z}$ is proportional to the identity one and $\underline{N}$ is zero.

VI - SOME ASPECTS RELATIVE TO THE IMPLEMENTATION OF THE PROPOSED ESTIMATORS

Some further points should be made regarding the use of the proposed estimators.

i) Up to now it has been supposed that all the available information was used by the various estimators.

It can however happen that some further information is available in the form of a parameter feasibility set S. A typical case is the non-negativity restriction for O-D components of $\underline{t}$. In the spirit of this paper, this information should be used in the estimation procedure, e.g. by minimizing objective functions (11), (18), (20) and (24) subject to admissibility constraints. It can be shown that if the imposed restrictions are respected by the true parameter vector then the estimates variance are smaller than that previously calculated.

ii) In real cases, true dispersion matrices are seldom, if ever, exactly known. Only more or less approximate estimates are available for them apart from the case in which they are "a priori" distribution parameters.

In this sense it is more correct to speak of Estimated Generalized Least Squares estimators. EGLS estimator properties coincide with GLS ones only asymptotically. However many Monte Carlo studies tend to support the conclusion that EGLS estimators are superior to LS ones, apart from the case of very slight heteroschedasticity, see for instance Goldfeld and Quant (1972).

iii) Calculation of the assignment matrix $\underline{\bar{A}}$ can follow two different approaches. If a proportional assignment model is used, e.g. all-or-nothing or stochastic network loading models, matrix $\underline{\bar{A}}$ can be calculated independentely from travel demand $\underline{t}$. If, however, stochastic or deterministic equilibrium model is used because it is deemed to give a better representation of real path choice behaviour and consequently less disperse estimates, then matrix $\underline{\bar{A}}$ itself is function of the travel demand characteristics to be estimated.

To the author's knowledge no general algorithms exist for solving problems (11), (18), (20) and (24) under equilibrium assignment. Algorithms proposed by Jornsten and Nguyen (1979), Gur et al. (1980), Nguyen (1983) can be adapted for solving the O-D matrix estimation problem with deterministic flow constraints, i.e. with a zero $\underline{N}$ matrix. Also a heuristic approach similar in spirit to that proposed by Willumsen (1980) and Nguyen (1982) could be adapted by iteratively calculating $\underline{\beta}$ or $\underline{t}$ vectors and then updating the assignment matrix. However no convergence proof was given for such an algorithm.

iv) In previous sections no reference was made to the proposed estimators expected values.

In general they are biased even if starting estimates $\underline{\bar{t}}$ or $\underline{\bar{\beta}}$ are unbiased except for the case of a linear model as in the case of direct O-D matrix estimation. Estimator biases, however, asymptotically tend to zero as the sample size increases.

This implies that, although large sample performance of proposed non-linear estimators are superior to those of other possible unbiased estimators based on sampling survey results only, for limited dimension samples their relative performances should be compared by risk or GMSE criteria via numerical simulation studies.

It should however be realized that almost every estimation technique for the parameters of a non-linear model gives rise to biased estimators, so that traffic flow information, if unbiased, should reduce such starting estimate biases and consequently improve both its first and second order statistical properties.

CONCLUSION

In this paper a class of travel demand parameter estimators was proposed based on the assumption that the combined use of different information sources produces "better" estimators.

In particular two cases were considered : a first in which "ad hoc" sample surveys and traffic flows are available and a second in which planner's "a priori" knowledge is combined with traffic flow based information. In both cases estimators were obtained for demand model parameters and O-D matrices. All deriving estimators, which are GLS or ML when based on experimental results only and Bayesian otherwise, can be obtained by solving an extremum problem with a quadratic in the demand objective function. Furthermore it has been shown that, under reasonable hypotheses, use

of traffic flows results in an improvement of demand parameter estimates even when only an approximate assignment model is used, provided that due allowance is made for assignment errors. Approximate dispersion matrix expressions, when meaningful, were given for various estimators confirming the variance reduction assumption. Also a small numerical calculation was performed to evaluate the combined effects of the significant parameters and results seem to confirm that a substantial reduction in estimate variances can be obtained.

APPENDIX

To obtain an order of magnitude of the variance reduction effect, the dispersion matrix for a two-stage estimator $\underline{\beta}_1$ of a gravity model parameters was calculated and compared with that of an estimator $\underline{\bar{\beta}}$ making use of sample surveys only.

In particular the small test network with four centroids shown in figure 1 was considered. It was assumed that the number of trips actually made between different zone pairs was a multinomial random variable with a mean value given by a gravity model :

$$t_{ij} = p_i q_j \exp(-\beta C_{ij}) + \gamma_{ij} \qquad (A1)$$

where the random residual γ_{ij} is a zero mean random variable.

Model residual variances were obtained as :

$$\text{Var}(\gamma_{ij}) = (\sum_j \bar{t}_{ij})\, \pi_{ij}\, (1-\pi_{ij}) \qquad (A2)$$

with $\bar{t}_{ij}$ representing the O-D pair mean value and π_{ij} the probability that a trip starting from zone i ends in zone j. The O-D matrix resulting from the gravity model is shown in figure 1.

It was assumed that surveys were carried out in the four zones with sampling rates a ranging from 5% to 20%. Under the hypothesis that trips originating in each zone were independentely sampled, estimated O-D matrix elements $\bar{t}_{ij}$ follow a multinomial distribution :

$$\bar{t}_{ij} = t_{ij} + \zeta_{ij} \qquad (A3)$$

$$E(\zeta_{ij}) = 0 \qquad \text{Var}(\zeta_{ij}) = (\sum_j t_{ij})\, \pi_{ij}\, (1-\pi_{ij})/\alpha \qquad (A4)$$

A starting estimate $\underline{\bar{\beta}}$ of the gravity model parameters p_i, q_j and β was obtained by using a GLS estimator solution of the problem :

$$\min_{\beta} (\bar{\underline{t}}-\underline{t}(\underline{\beta}))' \ \underline{M}^{-1} \ (\bar{\underline{t}}-\underline{t}(\underline{\beta})) \tag{A5}$$

The dispersion matrix of $\bar{\underline{\beta}}$ can be approximated by :

$$\underline{V} \doteq [J(\underline{\beta})' \ \underline{M}^{-1} \ J(\underline{\beta})]^{-1} \tag{A6}$$

where $\underline{M}$ is the sum of $\underline{G}$ and $\underline{Z}$ resulting from expression (3).

To obtain the dispersion matrix $\underline{B_1}$ of the dispersion matrix of the two stage estimators β_1 via expression (13), we still need to specify the flow dispersion matrix $\underline{W}$.

In this example no assignment errors exist however measurement error variances were assumed proportional to the square of true flows, through a varying coefficient K in order to evaluate effects of flow "imprecision" on the resulting estimates quality :

$$\text{var} (\bar{f}_i) = k f_i^2 \tag{A7}$$

In conclusion, the two stage estimator dispersion matrix $\underline{B_1}$ was calculated for each pair of sampling rate and flow variance values, both measured flows or two links, (6,5) and (5,6), and four links, (6,5), (5,6), (1,5) and (5,1).

The Mean Square Error for both $\bar{\underline{\beta}}$ and $\underline{\beta}$ was calculated as the trace of relative dispersion matrices. In table 1, MSE reduction as a percentage of the starting estimator $\bar{\underline{\beta}}$ MSE is reported.

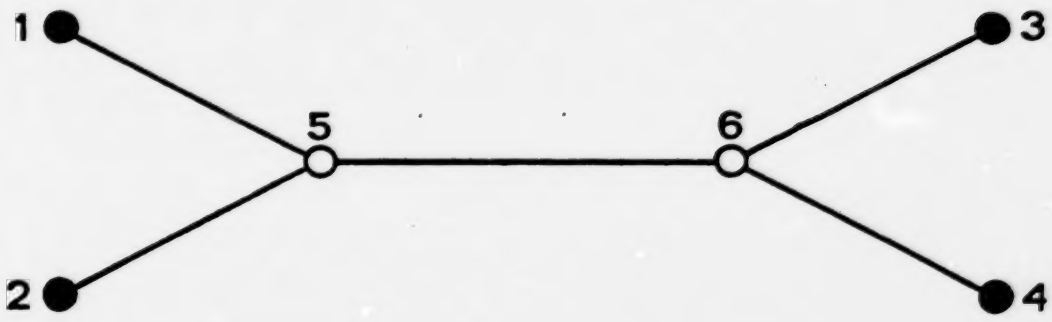
It can be seen that the variance reduction effect is stronger the lower the sampling rate and flow dispersions. Furthermore variations in MSE reduction due to sampling rates decrease for increasing flow dispersions.

In passing from two to four observed flows the MSE reduction percentage increases considerably, almost doubling its value.

From a practical point of view, the result is that a two stage estimator can bring considerable improvement for dispersed starting estimates, which often occur because of typically low sampling fractions, and for reasonable assignment models. In the example considered, the use of two flows, i.e. 20% of network links causes a reduction of approximately 35% in the MSE of model parameter estimates for typical values of the sampling fraction (0.05) and the variation coefficient of measurement errors (0.10).

Table 1 : MSE Reduction Percentage

	2 links				4 links			
K ^{a}	0,05	0,10	0,15	0,20	0,05	0,10	0,15	0,20
0,00	37	35	34	32	71	66	63	61
0,02	29	23	20	17	53	44	38	34
0,04	24	17	14	12	45	35	29	25
0,06	20	14	11	9	39	29	23	20
0,08	17	12	9	7	35	25	19	16
0,10	15	10	7	6	32	22	17	14
0,12	14	9	6	5	29	19	15	15
0,14	12	8	6	5	27	17	13	11
0,16	11	7	5	4	25	16	12	10
0,18	10	6	4	4	23	14	11	9
0,20	10	6	4	3	21	13	10	8



O \ D	1	2	3	4
1	182	74	45	22
2	221	243	89	45
3	100	67	182	55
4	201	134	221	182

Figure 1 - Test network

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